Brief description of RUN optimization algorithm

1 Updating solutions

The RUN algorithm uses a search mechanism (SM) based on the Runge Kutta method to update the position of current solution at each iteration, which is defined as,

\[
\begin{align*}
if \ rand < 0.5 \\
(\text{exploration phase}) \quad x_{n+1} &= (x_c + r \cdot SF \cdot g \cdot x_c) + SF \cdot SM + \mu \cdot \text{randn}.(x_m - x_c) \\
else \\
(\text{exploitation phase}) \quad x_{n+1} &= (x_m + r \cdot SF \cdot g \cdot x_m) + SF \cdot SM + \mu \cdot \text{randn}.(x_{r1} - x_{r2})
\end{align*}
\]

where \( r \) is an integer number, which is 1 or -1. \( g \) is a random number in the range [0, 2]. \( SF \) is an adaptive factor. where \( \mu \) is a random number. The formula of \( SM \) is defined in Appendix A.

The formula of \( SF \) is as follows:

\[
SF = 2.(0.5 - rand) \times f \tag{2}
\]

\[
f = a \times \exp \left( -b \times \text{rand} \times \left( \frac{i}{\text{Maxi}} \right) \right) \tag{2-1}
\]

where \( \text{Maxi} \) stands for the largest number of iterations. The formula of \( x_c \) and \( x_m \) are as follows:

\[
x_c = \varphi \times x_n + (1 - \varphi) \times x_{r1} \tag{3}
\]

\[
x_m = \varphi \times x_{best} + (1 - \varphi) \times x_{ibest} \tag{4}
\]

where \( \varphi \) is a random number in the range of (0,1). \( x_{best} \) is the best-so-far solution. \( x_{ibest} \) is the best position obtained at each iteration.

2. Enhanced solution quality (ESQ)

In the RUN algorithm, enhanced solution quality (ESQ) is employed to increase the quality of solutions and avoid local optima in each iteration. The following scheme is executed to create the solution \( x_{new2} \) by using the ESQ:

\[
if \ rand < 0.5 \\
if \ w < 1 \\
x_{new2} &= x_{new1} + r \cdot w \cdot |(x_{new1} - x_{avg}) + \text{randn}| \\
else \\
x_{new2} &= (x_{new1} - x_{avg}) + r \cdot w \cdot |u \cdot x_{new1} - x_{avg}) + \text{randn}| \\
end
\]

\[
w = \text{rand}(0, 2) \cdot \exp \left( -c \left( \frac{i}{\text{Maxi}} \right) \right) \tag{5-1}
\]
where \( \beta \) is a random number in the range of \([0, 1]\). \( \mathbf{c} \) is a random number, which is equal to \( 5 \times \mathbf{rand} \) in this study. \( \mathbf{r} \) is an integer number, which is 1, 0, or -1. \( x_{best} \) is the best solution explored so far.

The solution calculated in this part \((x_{new2})\) may not have better fitness than that of the current solution (i.e., \(f(x_{new2}) > f(x_n)\)). To have another chance for creating a good solution, another new solution \((x_{new3})\) is generated, which is defined as follows:

\[
\text{if } \mathbf{rand} < \mathbf{w} \\
x_{new3} = (x_{new2} - \mathbf{rand} x_{new2}) + SF.(\mathbf{rand} x_{RK} + (\mathbf{v} x_{b} - x_{new2}))
\]

where \( \mathbf{v} \) is a random number with a value of \(2 \times \mathbf{rand}\).

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**Algorithm 1.** The pseudo-code of RUN

**Stage 1. Initialization**
- Initialize \(a, b\)
- Generate the RUN population \(X_n (n = 1, 2, ..., N)\)
- Calculate the objective function of each member of population
- Determine the solutions \(x_w, x_b, \) and \(x_{best}\)

**Stage 2. RUN operators**
- for \(i = 1: Maxi\)
  - for \(n = 1 : N\)
    - for \(l = 1 : D\)
      - Calculate position \(x_{n+1,l}\) using Eq. 1
  - Enhance the solution quality
    - if \(\mathbf{rand} < 0.5\)
      - Calculate position \(x_{new2}\) using Eq. 5
      - if \(f(x_n) < f(x_{new2})\)
        - if \(\mathbf{rand} < \mathbf{w}\)
          - Calculate position \(x_{new3}\) using Eq. 6
    - end
  - end
- Update positions \(x_w, x_b\)
- end for
- Update position \(x_{best}\)
  - \(i = i+1\)
- end

**Stage 3.** return \(x_{best}\)
Appendix A:

The formula of $SM$ is defined as,

$$k_1 = \frac{1}{2\Delta x}(\text{rand} \times x_w - u \times x_h)$$

$$u = \text{round}(1 + \text{rand}) \times (1 - \text{rand})$$

$$k_2 = \frac{1}{2\Delta x}(\text{rand} \times (x_w + \text{rand}_1 \times k_1 \Delta x) - (u \times x_h + \text{rand}_2 \times k_1 \Delta x))$$

$$k_3 = \frac{1}{2\Delta x}(\text{rand} \times (x_w + \text{rand}_1 \times k_2 \Delta x) - (u \times x_h + \text{rand}_2 \times k_2 \Delta x))$$

$$k_4 = \frac{1}{2\Delta x}(\text{rand} \times (x_w + \text{rand}_1 \times k_3 \Delta x) - (u \times x_h + \text{rand}_2 \times k_3 \Delta x))$$

$$SM = \frac{1}{6}(x_{RR})\Delta x$$

$$x_{RR} = k_1 + 2 \times k_2 + 2 \times k_3 + k_4$$

where $\text{rand}_1$ and $\text{rand}_2$ are two random numbers in the range of $[0, 1]$. The formula of $\Delta x$ is defined as,

$$\Delta x = 2 \times \text{rand} \times |\text{Stp}|$$

$$\text{Stp} = \text{rand} \times \left((x_b - \text{rand} \times x_{avg}) + \gamma\right)$$

$$\gamma = \text{rand} \times (x_n - \text{rand} \times (u - l)) \times \exp(-4 \times \frac{i}{\text{Maxl}})$$

In this study, $x_w$ and $x_h$ are determined by the following:

```plaintext
if f(x_n) < f(x_{bl})
    x_b = x_n
    x_w = x_{bi}
else
    x_b = x_{bi}
    x_w = x_n
end
```