Brief description of RUN optimization algorithm

1 Updating solutions

The RUN algorithm uses a search mechanism (SM) based on the Runge Kutta method to update the

$$
\begin{aligned}
&\boldsymbol{if}\ \ rand < 0.5\\
&\qquad \textbf{(exploration phase)}\\
&\qquad x_{n+1} = (x_c + r.SF.g.x_c) + SF.SM + \mu.randn.(x_m - x_c)\\
&\qquad \boldsymbol{else}\\
&\qquad \textbf{(exploitation phase)}\\
&\qquad x_{n+1} = (x_m + r.SF.g.x_m) + SF.SM + \mu.randn.(x_{r1} - x_{r2})\\
&\boldsymbol{end}
\end{aligned} \tag{1}
$$

position of current solution at each iteration, which is defined as,

where r is an integer number, which is 1 or -1. g is a random number in the range [0, 2]. SF is an adaptive factor. where μ is a random number. The formula of SM is defined in Appendix A.

The formula of SF is as follows:

$$SF = 2.(0.5 - rand) \times f \tag{2}$$

$$f = a \times exp\left(-b \times rand \times \left(\frac{i}{Maxi}\right)\right) \tag{2-1}$$

where $Maxi$ stands for the largest number of iterations. The formula of x_c and x_m are as follows:

$$x_c = \varphi \times x_n + (1 - \varphi) \times x_{r1} \tag{3}$$

$$x_m = \varphi \times x_{best} + (1 - \varphi) \times x_{lbest} \tag{4}$$

where φ is a random number in the range of (0,1). x_{best} is the best-so-far solution. x_{lbest} is the best position obtained at each iteration.

2. Enhanced solution quality (ESQ)

In the RUN algorithm, enhanced solution quality (ESQ) is employed to increase the quality of solutions and avoid local optima in each iteration. The following scheme is executed to create the solution (x_{new2}) by using the ESQ:

$$
\begin{aligned}
&\boldsymbol{if}\ \ rand < 0.5\\
&\quad \boldsymbol{if}\ \ w < 1\\
&\qquad x_{new2} = x_{new1} + r.w.\left|\left(x_{new1} - x_{avg}\right) + randn\right|\\
&\quad \boldsymbol{else}\\
&\qquad x_{new2} = (x_{new1} - x_{avg}) + r.w.\left|\left(u.x_{new1} - x_{avg}\right) + randn\right|\\
&\quad \boldsymbol{end}\\
&\boldsymbol{end}
\end{aligned} \tag{5}
$$

$$w = rand(0, 2).exp\left(-c\left(\frac{i}{Maxi}\right)\right) \tag{5-1}$$

$$x_{avg} = \frac{x_{r1} + x_{r2} + x_{r3}}{3} \tag{5-2}$$

$$x_{new1} = \beta \times x_{avg} + (1 - \beta) \times x_{best} \tag{5-3}$$

where β is a random number in the range of [0, 1]. c is a random number, which is equal to $5\times rand$ in this study. r is an integer number, which is 1, 0, or -1. x_{best} is the best solution explored so far.

The solution calculated in this part (x_{new2}) may not have better fitness than that of the current solution (i.e., $f(x_{new2}) > f(x_n)$). To have another chance for creating a good solution, another new solution (x_{new3}) is generated, which is defined as follows:

if rand< $\boldsymbol{w}$

$$x_{new3} = (x_{new2} - rand.x_{new2}) + SF.(rand.x_{RK} + (v.x_b - x_{new2})) \tag{6}$$

end

where v is a random number with a value of $2 \times rand$.

Algorithm 1. The pseudo-code of RUN

```
Stage 1. Initialization
Initialize a, b
Generate the RUN population X_n (n = 1, 2, …, N)
Calculate the objective function of each member of population
Determine the solutions x_w, x_b, and x_best
Stage 2. RUN operators
  for i= 1: Maxi
     for n = 1 : N
        for l = 1 : D
          Calculate position x_{n+1,l} using Eq. 1
       end for
       Enhance the solution quality
        if rand < 0.5
         Calculate position x_new2 using Eq. 5
           if f(x_n) < f(x_new2)
            if rand< w
                 Calculate position x_new3 using Eq. 6
             end
           end
        end
      Update positions x_w and x_b
     end for
     Update position x_best
    i=i+1
  end
Stage 3. return x_best
```

Appendix A:

The formula of SM is defined as,

$$k_1 = \frac{1}{2\Delta x}(rand \times x_{\mathrm{w}} - u \times x_{\mathrm{b}})$$

$$u = round(1 + rand) \times (1 - rand)$$

$$k_2 = \frac{1}{2\Delta x}(rand.(x_{\mathrm{w}} + rand_1.k_1.\Delta x) - (u.x_{\mathrm{b}} + rand_2.k_1.\Delta x))$$

$$k_3 = \frac{1}{2\Delta x}(rand.(x_{\mathrm{w}} + rand_1.\left(\frac{1}{2}k_2\right).\Delta x) - (u.x_{\mathrm{b}} + rand_2.\left(\frac{1}{2}k_2\right).\Delta x))$$

$$k_4 = \frac{1}{2\Delta x}(rand.(x_{\mathrm{w}} + rand_1.k_3.\Delta x) - (u.x_{\mathrm{b}} + rand_2.k_3.\Delta x))$$

$$SM = \frac{1}{6}(x_{RK})\Delta x$$

$$x_{RK} = k_1 + 2 \times k_2 + 2 \times k_3 + k_4$$

where $rand_1$and $rand_2$ are two random numbers in the range of [0, 1]. The formula of Δx is defined as,

$$\Delta x = 2 \times rand \times |Stp|$$

$$Stp = rand \times \left(\left(x_b - rand \times x_{avg}\right) + \gamma\right)$$

$$\gamma = rand \times (x_n - rand \times (u - l)) \times \exp(-4 \times \frac{i}{Maxi})$$

In this study, x_{w} and x_{b} are determined by the following:

$$\begin{array}{l} \boldsymbol{if}\ f(x_n) < f(x_{bi}) \\ \qquad x_b = x_n \\ \qquad x_w = x_{bi} \\ \boldsymbol{else} \\ \qquad x_b = x_{bi} \\ \qquad x_w = x_n \\ \boldsymbol{end} \end{array}$$